Performance Benchmarks of Quantum Simulators, Noise Models and Error Correction

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*Abstract*—Quantum Computing is evolving at a pace that was never seen before. Corporation and Government realize the immense potential in the technology to radicalize the world we live in. In this fast-paced environment, this study provides a comparison of the existing quantum simulators with respect to several key performance benchmarks. We will discuss the importance each of the performance metrics and how the various error correction methods help us in mitigating these errors. We also discuss briefly about the various noise models which can be efficiently used to simulate in the classical computers.

Keywords—Quantum Computing, Quantum Simulators, Noise Models, Error Correction Techniques

# Functions of a Quantum Simulator

Quantum simulators [1] are software programs that run on classical computers and act as the target machine, making it possible to run and test quantum programs in an environment that predicts how qubits will react to different operations. The quantum simulator is responsible for providing implementations of quantum operations for an algorithm. This includes primitive operations such as H, CNOT, and Measure, as well as qubit management and tracking. The Quantum Development Kit includes different classes of quantum simulators representing different ways of simulating the same quantum algorithm. I have listed below some of the Quantum Simulators available in the industry and are accessible through cloud.

* IBM’s Qiskit
* Google’s Cirq
* Amazon’s AWS Bracket
* Microsoft’s Q# and Azure Quantum
* Rigetti’s Forest
* Xanadu’s Pennylane

# DIVENCENZO CRITERIA

## For Quantum computation

* A Scalable Physical System with well characterized qubits
* The ability to initialize the state of the qubits to a simple fiducial state
* Long Relevant Decoherence times
* A “UNIVERSAL” set of quantum gates
* A Qubit-specific measurement capability

## For Quantum Communication

* The Ability to interconvert stationary and flying qubits
* The ability to faithfully transmit flying qubits between specified locations

# Quantum Computing Roadmap

According to google, this is the roadmap of the Quantum Information research for this decade. Currently, most of the research is happening in the first 2 points

* Implement Error Correction
* Show Error Correction gets better with more qubits
* Make 1 logical qubit with endless coherency
* Make 2 logical qubits with 2-qubit operations
* Tile Thousands of logical qubits

# Quantum simulator performance metric Terminologies

## Circuit Depth

The length of the longest path from the input to output or the minimum amount of time taken to execute the circuit (assuming every gate operation are performed in same time-step) [2]

## Circuit Width

The circuit width is the total number of input qubits and bits used in the circuit. [2]

## Quantum Volume

It is defined as the average performance on a set of random circuits. If a quantum computer can execute an algorithm successfully (error-free) with n qubits, its quantum volume is 2^n. [2]

## Quantum Circuits and Programs

A quantum circuit is a computational routine consisting of an ordered sequence of quantum operations including gates, measurements, and resets on quantum data (qubits) and concurrent real-time classical computation. Data flows between the quantum operations and the real-time classical compute so that the classical compute can incorporate measurement results and the quantum operations may be conditioned upon or parameterized by data from the real-time classical compute. Here real-time means within the coherence time of the

qubits. [2]

## Circuit Layer Operations per second (CLOPS)

Circuit Layer Operations per Second (CLOPS) is a measure correlated with how many QV circuits a QPU can execute per unit of time. That simple statement hides a wealth of choice about the possible circuit families and the execution context. Here, we pursue a holistic speed benchmark of a typical application. To faithfully model real-world use, we deem it essential to capture interaction time with the run-time environment that invokes the circuits. This attempts to avoid a pitfall seen in some synthetic benchmarks that characterize classical systems by their instruction clock rate without considering the effects of data transfers between CPU, cache, and main memory. [2]

# Performance Metrics

Currently, the following performance metrics are used to evaluate a Quantum Computers [1]

1. Scale
2. Quality
3. Speed

* Scale: Measured by the number of qubits. Indicates the amount of information we can encode in a quantum system
* Quality: Measured by Quantum Volume which indicates quality of circuits and how faithfully circuits are implemented in hardware
* Speed: Measured by CLOPS (Circuit Layer Operations per Second) which indicates how many circuits can run on hardware in each time.

Diagram

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FIG. 1. Benchmarking pyramid showing how quality and

speed can be benchmarked. Higher-level benchmarks capture

more complexity but less specificity. There may be tradeoffs

between the two faces of the pyramid [1].

# Quantum Noise and decoherance

Noise describes all the things that can cause the quantum computers to malfunction. The source of quantum noise includes electromagnetic signals, earth’s magnetic field, due to which the qubit states inherently change slightly from its original state [4]

This change in the qubit state due to environmental factors is called decoherence of qubits

# Quantum Error Correction

## Barriers to Quantum Error Correction

1. Measurement of error destroys superpositions.
2. No-cloning theorem prevents repetition.
3. Must correct multiple types of errors (e.g., bit flip and phase errors).
4. How can we correct continuous errors and decoherence?

## Types of Quantum Errors

There are the common types of errors that occur in quantum computers.

1. Bit Flip Errors
2. Phase Flip Errors
3. Complete Dephasing Errors
4. Rotation Errors

Bit Flip X: X⎟0〉 = ⎟1〉, X⎟1〉 = ⎟0〉

Phase Flip Z: Z⎟0〉 = ⎟0〉, Z⎟1〉 = -⎟1〉

Complete dephasing: ρ → (ρ + ZρZ†)/2 (decoherence)

Rotation: Rθ⎟0〉 = ⎟0〉, Rθ⎟1〉 = eiθ⎟1〉

A distance measure [7] quantifies the extent to which two quantum states behave in the same way. While these distance measures are usually given by certain mathematical expressions, they often possess a simple operational meaning, i.e., they are related to the problem of distinguishing two systems. These distance metrics are used to compare two qubit states to quantify the errors occurring in the system

## Bit flip error correction / Error Correction via Repetition

Graphical user interface

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FIG. 2. Bit Flip Error Correction Algorithm

Graphical user interface

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FIG. 3. Bit Flip Error Correction Qubit Comparison

Below is the circuit model for achieving the error corrected logical qubit via repetition

Diagram, schematic

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FIG. 4. Bit Flip Error Correction Qiskit Implementation

Code distance vs ln(Logical error probability) is given below

Chart, scatter chart

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FIG. 5. Code distance vs ln(Logical error probability)

## Phase flip Errors

Hadamard transform H exchanges bit flip and phase errors:

H (α⎟0〉 + β⎟1〉) = α⎟+〉 + β⎟-〉

X⎟+〉 = ⎟+〉, X⎟-〉 = -⎟-〉 (acts like phase flip)

Z⎟+〉 = ⎟-〉, Z⎟-〉 = ⎟+〉 (acts like bit flip)

Repetition code in Hadamard basis corrects a phase error.

α⎟+〉 + β⎟-〉 → α⎟000〉 + β⎟000〉

## Bit and Phase flip error correction / Nine Qubit Codes

To correct both bit flips and phase flips, use both codes at once:

α⎟0〉 + β⎟1〉 → α(⎟000〉 + ⎟111〉)⊗3 + β(⎟000〉 - ⎟111〉)⊗3

Repetition 000, 111 corrects a bit flip error, repetition of phase +++, --- corrects a phase error. This code corrects a bit flip and a phase, so it also corrects a Y error:

Y = iXZ: Y⎟0〉 = i⎟1〉, Y⎟1〉 = -i⎟0〉 (Global Phase irrelevant)

## Correcting Continuous Rotations

How does error correction affect a state with a continuous rotation on it? [8]

Rθ(k)⎟ψ〉 = cos (θ/2)⎟ψ〉 - i sin (θ/2) Z(k)⎟ψ〉

cos (θ/2)⎟ψ〉⎟I〉 - i sin (θ/2) Z(k)⎟ψ〉 ⎟Z(k)〉 (Error Syndrome)

Measuring the error syndrome collapses the state:

Prob. cos2 (θ/2): ⎟ψ〉 (no correction needed)

Prob. sin2 (θ/2): Z(k)⎟ψ〉 (corrected with Z(k))

## Correcting All Single-Qubit Errors

Theorem: If a quantum error-correcting code (QECC) corrects errors A and B, it also corrects αA + βB. Any 2x2 matrix can be written as αI + βX + γY + δZ. [8]

A general single-qubit error ρ → Σ Ak ρ Ak† acts like a mixture of ⎟ψ〉 → Ak⎟ψ〉, and Ak is a 2x2 matrix.

Any QECC that corrects the single-qubit errors X, Y, and Z (plus I) corrects every single-qubit error.

Correcting all t-qubit X, Y, Z on t qubits (plus I) corrects all t-qubit errors.

## Shor Code Implementation

Shor code is used in error correction of both the bit flip errors and phase flip errors. [10]

Diagram

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FIG. 6. Circuit Diagram of the Shor Code

# Quantum Metric Simulations

## Quantum Volume Simulation

The following algorithms are simulated and measured for Quantum Volume. These are the results. [12]

A picture containing chart

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FIG. 7. Quantum Volume Simulation 1

Chart, waterfall chart

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FIG. 8. Quantum Volume Simulation 2

Chart, waterfall chart

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FIG. 8. Quantum Volume Simulation 3

## QFT Simulations and evaluating Performance metrics – Results

Parameters

With Noise model:

Single Qubit Depolarization Error: 0.3%

Two Qubit Error: 3%

Shots = 100

Chart, bar chart

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## FIG 9. QFT Simulations and evaluating Performance metrics – Results

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